Exercise 1. A rigid dualizing complex over $\mathcal{D}_X$ is a dualizing complex $\mathcal{R}$ together with a rigidifying isomorphism
\[ \rho : \mathcal{R} \to \mathcal{R} \mathcal{H} \mathcal{m}_{\mathcal{D}_X \otimes \mathcal{D}_X}(\mathcal{D}_X, \mathcal{R} \otimes \mathcal{R}). \]
(We remark that rigid dualizing complexes are unique up to unique isomorphism.) Show that $\mathcal{D}_X[2 \dim X]$ is rigid. [Hint: Let $\Delta : X \to X \times X$ be the diagonal morphism and use that $\Delta_*(\Delta_*\mathcal{O}_X) \cong \Delta_*(\Delta_*\mathcal{O}_X) \cong \Delta_*(\mathcal{O}_X).$]

Exercise 2. Let $\mathcal{M}$ be an integrable connection, i.e. an $\mathcal{O}_X$-coherent $\mathcal{D}_X$-module. Show that $\mathcal{D}\mathcal{M} \cong \mathcal{H} \mathcal{m}_{\mathcal{O}_X}(\mathcal{M}, \mathcal{O}_X)$.

Exercise 3. Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module with $\text{Ch}(\mathcal{M}) \subseteq X = T^*_XX$. Show that $\mathcal{M}$ is an integrable connection.

Exercise 4. Let $i : Z \hookrightarrow X$ be a closed embedding. Let
\[ i^*(T^*X) = Z \otimes_X T^*X \]
be the natural morphisms induced by $i$. Let $\mathcal{M}$ be a coherent $\mathcal{D}_Z$-module. Show that
\[ \text{Ch}(i_*\mathcal{M}) = \varpi(\rho^{-1}\text{Ch}(\mathcal{M})). \]
Deduce that $\mathcal{M}$ is holonomic if and only if $i_*\mathcal{M}$ is.

Exercise 5. Show that for any affine open subset $U$ of $X$ the ring $\mathcal{D}_X(U)$ has left and right global dimension $\dim X$. Deduce that any $\mathcal{M} \in \text{Mod}_{\mathcal{D}_X}$ has a locally projective resolution of length at most $\dim X$.

Exercise 6. Show that there exists a canonical morphism of functors
\[ f_! \to f_* : D^b_{\text{hol}}(\mathcal{D}_X) \to D^b_{\text{hol}}(\mathcal{D}_Y). \]